Optical solitons in resonant and nonresonant nonlinear media in the presence of perturbations

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We studied the optical solitons in nonlinear resonant and nonresonant media in the presence of perturbations, assuming that the transient effects are stimulated by the light scanning beam. We treated a slight deviation from the exact necessary condition for the soliton existence $(2\beta\nu=1)$, as a small perturbation for the integrable system, studying its influence upon the soliton propagation conditions. The approximation is constructed by the help of an algebraic version of the soliton perturbation theory using a Riemann boundary problem in connection with the inverse scattering method. We have obtained the soliton equation and we have solved it in the presence of a small perturbation in the adiabatic approximation. In this case we have demonstrated that for a Lorentz profile line the amplitude of the soliton remains unchanged, the only effect of the perturbation results in a phase shift.

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I. INTRODUCTION

When a short optical pulse propagates through a dispersive nonlinear optical medium, the spectral content of the pulse can become modified because of the nonlinear optical process of self-phase modulation effect. The shape of the pulse can modify because of the propagation effects, such as dispersion of the group velocity within the medium [1].

Under certain conditions it is possible for the effects of the group velocity dispersion to completely compensate the effects of the self-modulation, so that optical pulses can propagate through a dispersive, nonlinear optical medium with an invariant shape. These pulses are known as optical solitons.

The study of the propagation of the light pulses in media with nonresonant nonlinearities establishes the effects of the self-focusing and anomalous dispersion on the pulse evolution. The formation of solitons in such combined media was first demonstrated [2–4] by considering the scanning of a bidimensional light beam, normally incident onto the surface of a nonlinear medium. Under specific conditions, the light soliton emerges, having the properties of a traveling waveguide channel and of a 2π pulse.

Owing to the analogy between the equations describing the propagation of the incidental beams [3,5,6] and of the propagating pulses, we can study the evolution of the soliton pulses in combined media. The formal similitude of the equations allows the application of the results obtained when studying the incidental pulses, to the pulses propagating in the nonlinear medium by direct correspondence of the notations.

We studied the optical solitons in nonlinear resonant and nonresonant media in the presence of perturbations, assuming that the transient effects are stimulated by the scanning beam. Using the self-induced transparency theory, the necessary condition for the soliton existence is $2\beta\nu=1$, where ν is the adimensional scanning speed and β is the parameter of the Kerr-type nonlinearity. This condition expresses a balance between different competing factors, such as the diffraction divergence, the self-focusing, etc.

From the mathematical point of view, this condition represents the existence criterion of the Lax representation. In practice, the exact relationships between the parameters of the radiation and those of the medium may present a slight deviation. Such a deviation can be treated as a perturbation of the integrable system considered, thus the problem of the perturbed solitons investigation appears. Extensions of the perturbed solitons theory were made by Kaup [7] and Karpman [8].

The present study is important because of several reasons. First, the study is of great interest and leads to a sufficiently general approximation that can also be used in other almost integrable systems. Thus, we demonstrate the efficiency of using the Riemann problem when analyzing the soliton perturbation. On the other hand, many possible applications and theoretical studies are permitted. For example, a similar analytical approximation can be used in the case of the propagation of the pseudosoliton pulses in dielectric waveguides (optical fibers) having a resonant nonlinearity associated with a cubic nonlinearity.

In this study we have formulated the Riemann boundary problem in connection with the inverse scattering method. Then we present the necessary elements of the soliton perturbation theory within the framework of the Riemann problem.

II. THE RIEMANN PROBLEM FOR SOLITONS

The equations system describing the stationary distribution of the scanning beam propagating in a Kerr-type resonant medium, written in adimensional variables is [2,3]

$$\frac{de}{dz}+i\frac{d^2e}{dz^2}-\nu\frac{de}{dx}+2i\beta^2|e|^2e+\langle\rho\rangle=0, \tag{1.1}$$

$$\nu\frac{d\rho}{dx}-i\delta\rho+en=0, \tag{1.2}$$

$$\nu\frac{dn}{dx}-\frac{1}{2}(e\bar{\rho}+\bar{e}\rho)=0, \tag{1.3}$$

where e is the slowly varying complex envelope of the light beam, ρ is the polarizability of the medium, and n is the population difference of the medium levels resonant with the incidental light.

The angular brackets in Eq. (1.1) signify an averaging over the frequency detuning $\delta\approx(\omega-\omega_{21})$ inside the inhomogenously broadened line,

$$\langle\rho\rangle=\int_{-\infty}^{\infty}\rho(\delta)g(\delta)d\delta, \tag{2}$$

where $g(\delta)$ is the profile of the line.

The normalization condition for the distribution $g(\delta)$ is

$$\int_{-\infty}^{\infty}g(\delta)d\delta=1. \tag{3}$$

ω is the frequency of the incidental light beam and ω_{21} is the resonant transition frequency. The Ox axis is situated on the surface of the nonlinear medium and the Oz axis is oriented towards the center of the medium.

Eqs. (1.1),(1.2),(1.3) are integrable and admit a soliton solution if the following condition is fulfilled [2]:

$$2\beta\nu=1. \tag{4}$$

The zero curvature representation

$$\frac{dU}{dz}-\frac{dV}{dx}+[U,V]=0 \tag{5}$$

is equivalent to Eqs. (1) and (3) and it is given by the U and V matrices of the form

$$U=\beta(e\sigma_+-\bar{e}\sigma_-)-i\zeta\sigma_3\equiv U_0-i\zeta\sigma_3, \tag{6}$$

$$\begin{aligned}V=-\beta\Bigg\{&\left[ie_x+\left(2\zeta-\frac{1}{2\beta}\right)e-\frac{i}{2}\left\langle\frac{\rho}{\zeta+\beta\delta}\right\rangle\right]\sigma_+\\&+\left[i\bar{e}_x-\left(2\zeta-\frac{1}{2\beta}\right)\bar{e}-\frac{i}{2}\left\langle\frac{\bar{\rho}}{\zeta+\beta\delta}\right\rangle\right]\sigma_-\\&-i\left[\frac{1}{2}\left\langle\frac{n}{\zeta+\beta\delta}\right\rangle-\beta|e|^2+\frac{\zeta}{\beta}\left(2\zeta-\frac{1}{2\beta}\right)\right]\sigma_3\Bigg\},\end{aligned} \tag{7}$$

where ζ is the spectral parameter, σ_3, $\sigma_\pm=\sigma_1\pm i\sigma_2$ are the Pauli matrices, and $\bar{e}$ is the complex conjugate of e.

For solving Eqs. (1) by the inverse scattering method [8], we analyze the spectral problem

$$\frac{d\Phi(x,\zeta)}{dx}-U(x,\zeta)\Phi(x,\zeta)=0. \tag{8}$$

We denote by $T_\pm(x,\zeta)$ the Jost solutions matrix of Eq. (8), satisfying the conditions

$$T_\pm(x,\zeta)\xrightarrow[x\to\pm\infty]{}J(\zeta,x),\quad J(\zeta,x)=\exp(-i\zeta\sigma_3x). \tag{9}$$

Consequently, the scattering matrix $S(\zeta)$ is

$$T_-(x,\zeta)=T_+(x,\zeta)S(\zeta), \tag{10}$$

where

$$S(\zeta)=\begin{bmatrix}a(\zeta)&-\bar{b}(\zeta)\\b(\zeta)&\bar{a}(\zeta)\end{bmatrix},\quad \det S(\zeta)=1. \tag{11}$$

Let us introduce the matrices $\Psi_\pm=T_\pm J^{-1}$ and separate the two columns

$$\Psi_\pm(x,\zeta)=(\Psi_\pm^{(1)}(x,\zeta),\Psi_\pm^{(2)}(x,\zeta)). \tag{12}$$

Considering that the electromagnetic field $e(x)$ becomes zero sufficiently fast when $|x|\to\infty$, the columns $\Psi_-^{(1)}$ and $\Psi_+^{(2)}$ are analytical in the upper semiplane of the complex plane ζ and the columns $\Psi_-^{(2)}$ and $\Psi_+^{(1)}$ are analytical in the lower semiplane.

We define two new matrices $\theta(x,\zeta)$ and $\bar{\theta}(x,\zeta)$:

$$\theta(x,\zeta)=(\Psi_-^{(1)}(x,\zeta),\Psi_+^{(2)}(x,\zeta))$$

and (13)

$$\bar{\theta}(x,\zeta)=(\Psi_+^{(1)}(x,\zeta),\Psi_-^{(2)}(x,\zeta)),$$

fulfilling the conditions:

$$\det\theta(x,\zeta)=a(\zeta),$$

$$\det\bar{\theta}(x,\zeta)=\bar{a}(\zeta),$$

$$\theta(x,\zeta)\xrightarrow[x\to\pm\infty]{}J(\zeta,x)\theta_\pm(\zeta)J^{-1}(\zeta,x),$$

$$\bar{\theta}(x,\zeta)\xrightarrow[x\to\pm\infty]{}J(\zeta,x)\bar{\theta}_\pm(\zeta)J^{-1}(\zeta,x), \tag{14}$$

where

$$\theta_+(\zeta)=\begin{bmatrix}a&0\\b&1\end{bmatrix},\quad \theta_-(\zeta)=\begin{bmatrix}1&\bar{b}\\0&a\end{bmatrix},$$

$$\bar{\theta}_+(\zeta)=\begin{bmatrix}1&-\bar{b}\\0&\bar{a}\end{bmatrix},\quad \bar{\theta}_-(\zeta)=\begin{bmatrix}\bar{a}&0\\-b&1\end{bmatrix}. \tag{15}$$

The matrices $\theta(x,\zeta)$ and $\bar{\theta}(x,\zeta)$ are connected to the scattering matrix as follows:

$$\theta_+(\zeta)=S(\zeta)\theta_-(\zeta),\quad \bar{\theta}_+(\zeta)=S(\zeta)\bar{\theta}_-(\zeta). \tag{16}$$

Let ζ_j and $\bar{\zeta}_j$ be the zeros of $a(\zeta)$ and of $\bar{a}(\zeta)$, respectively, for $j=1,...,\bar{N}$. In this case, the columns θ and $\bar{\theta}$ are proportional,

$$\theta^{(1)}(x,\zeta_j)=\gamma_j(x)\theta^{(2)}(x,\zeta_j),$$

$$\gamma_j(x)=\gamma_j\exp(2i\zeta_jx),\quad \mathrm{Im}\,\zeta_j>0,\ \gamma_j\in\mathbf{C},$$

$$\bar{\theta}^{(2)}(x,\bar{\zeta}_j)=-\bar{\gamma}_j(x)\bar{\theta}^{(1)}(x,\bar{\zeta}_j),$$

$$\bar{\gamma}_j(x)=\bar{\gamma}_j\exp(-2i\bar{\zeta}_jx),\quad \mathrm{Im}\,\bar{\zeta}_j<0,\ \bar{\gamma}_j\in\mathbf{C}. \tag{17}$$

The set $b(\xi)$ ($\xi=\mathrm{Re}\,\zeta$), ζ_j, $\bar{\zeta}_j$, γ_j, and $\bar{\gamma}_j$ represent the scattering data.

Further on we present a method leading to the reconstruction of the potential $U_0(x)$ from the scattering data

$$U_0(x)=\beta\begin{bmatrix}0 & e\\ -\overline{e} & 0\end{bmatrix}. \tag{18}$$

From relation (13) it follows that

$$\overline{\theta}^{+}(x,\xi)\,\theta(x,\xi)=G(x,\xi), \tag{19}$$

where

$$G(x,\xi)=J(\xi x)G(\xi)J^{-1}(\xi x),$$

$$G(\xi)=\begin{bmatrix}1 & \overline{b(\xi)}\\ b(\xi) & 1\end{bmatrix},\quad \overline{\theta}^{+}=\det\overline{\theta\theta}^{-1}. \tag{20}$$

Equation (19) represents the matricial Riemann problem [i.e., the $G(x,\zeta)$ factorization, defined for real ζ, under the conditions that θ and $\overline{\theta}$ admit analytical prolongation in the upper and lower semiplanes of the ζ plane, respectively]. An analytical solution of the Riemann problem is of the form

$$\theta(x,\zeta)=I-\sum_{j=1}^{N}(\zeta_j-\zeta)^{-1}\frac{\overline{\theta}(x,\overline{\zeta_j})}{\dot{\overline{a}}(\overline{\zeta_j})}+\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{d\xi}{\xi-\zeta}\overline{\theta}(x,\xi)\overline{\rho}(x,\xi),\quad \operatorname{Im}\zeta>0$$

$$\overline{\theta}(x,\zeta)=I-\sum_{j=1}^{N}(\zeta_j-\zeta)^{-1}\frac{\theta(x,\zeta_j)}{\dot{a}(\zeta_j)}-\frac{1}{2\pi i}\times\int_{-\infty}^{\infty}\frac{d\xi}{\xi-\zeta}\theta(x,\xi)\rho^{+}(x,\xi),\quad \operatorname{Im}\zeta<0 \tag{21}$$

where

$$\dot{a}(\zeta_j)=(d/d\zeta)(a(\zeta))_{\zeta=\zeta_j},\quad \dot{\overline{a}}(\overline{\zeta_j})=(d/d_{\zeta})(\overline{a}(\zeta))_{\zeta=\overline{\zeta}_j},$$

$$\rho^{+}(x,\xi)=\frac{G^{+}(x,\xi)-I}{a(\xi)},\quad \overline{\rho}(x,\xi)=\frac{G(x,\xi)-I}{\overline{a}(\xi)} \tag{22}$$

and I is the identity matrix. Now we shall express the potential $U_0(x)$ as a function of $\overline{\theta}(x,\zeta)$.

This matrix admits the following asymptotic expansion [9]:

$$\overline{\theta}(x,\zeta)=I+\frac{1}{2i\zeta}\Omega(x)+O(|\zeta|^{-1}). \tag{23}$$

Then

$$U_0(x)=\frac{1}{2}[\sigma_3,\Omega(x)]\quad\text{or}\quad \beta e(x)=\Omega(x)_{12} \tag{24}$$

A. *z*-axis evolution of the scattering matrix

We consider the evolution equation of the form

$$\frac{d\Phi}{dz}-V\Phi=0. \tag{25}$$

We take the solution of Eq. (25) of the form

$$\Phi=\theta Jh(z), \tag{26}$$

where $h(z)$ is a function to be determined. Setting $x\to-\infty$ into Eq. (14) we obtained

$$\Phi\to J\theta_{-}h(z),$$

$$V\to V_{-}=\frac{i}{2}\left[-\beta\left\langle\frac{1}{\zeta+\beta\delta}\right\rangle+2\zeta\left(2\zeta-\frac{1}{2\beta}\right)\right]\sigma_3 \equiv\frac{i}{2}(\omega_1+i\omega_2)\sigma_3. \tag{27}$$

By introducing Eqs. (27) in Eq. (25) we finally get the evolution equation of the form

$$\frac{dh}{dz}=\left(\theta_{-}^{-1}V_{-}\theta_{-}-\theta_{-}^{-1}\frac{d\theta_{-}}{dz}\right)h. \tag{28}$$

For $x\to+\infty$, using Eq. (14) we obtain

$$\Phi\to JS\theta_{-}h(z). \tag{29}$$

Introducing Eq. (28) in Eq. (27) with the condition

$$\lim_{x\to-\infty}V=\lim_{x\to+\infty}V, \tag{30}$$

we get

$$\frac{dS}{dz}=[V_{-},S]. \tag{31}$$

From Eq. (31) it results

$$\frac{da}{dz}=0,\quad \frac{db}{dz}+i(\omega_1+i\omega_2)b=0. \tag{32}$$

The features of the discrete spectrum obey the following z evolution equations:

$$\frac{d\zeta_j}{dz}=0,$$

$$\frac{d\gamma_j}{dz}+i[\omega_1(\zeta_j)+i\omega_2(\zeta_j)]\gamma_j=0. \tag{33}$$

B. Soliton solution for the slowly varying complex envelope of the electric field

Taking $N=1$, $\rho=\overline{\rho}=0$, and by help of the relation (17) we can reduce the system (21) to the following form:

$$\theta_s(x,\zeta)=I-\frac{\zeta_1-\overline{\zeta_1}}{\zeta-\overline{\zeta_1}}\overline{\theta_s}(x,\overline{\zeta_1})\overline{F_1}(x),$$

$$\overline{\theta_s}(x,\zeta)=I-\frac{\overline{\zeta_1}-\zeta_1}{\zeta-\zeta_1}\theta_s(x,\zeta_1)F_1(x), \tag{34}$$

where we denoted by θ_s the corresponding soliton matrix in this particular case and

$$F_1(x)=\begin{bmatrix}0 & \gamma_1^{-1}(x)\\ \gamma_1(x) & 0\end{bmatrix}\equiv\begin{bmatrix}0 & \exp(y-i\vartheta)\\ \exp(-y+i\vartheta) & 0\end{bmatrix},$$

$$\bar{F}_1(x)=-\begin{bmatrix}0 & \bar{\gamma}_1(x)\\ \bar{\gamma}_1^{-1}(x) & 0\end{bmatrix}\equiv-\begin{bmatrix}0 & \exp(-y-i\vartheta)\\ \exp(y+i\vartheta) & 0\end{bmatrix}. \quad (35)$$

We denoted by

$$\gamma_1=\exp\{i[\omega_1(\zeta_1)+i\omega_2(\zeta_1)]z+i\varphi_0+x_0\},\quad \zeta_1=\xi_1+i\eta_1$$

$$y=2\eta_1(x-\chi),\quad \vartheta=(\xi_1/\eta_1)y+K$$

$$K=[(\xi_1/\eta_1)\omega_2-\omega_1]z+(\xi_1/\eta_1)x_0+\varphi_0+\frac{\pi}{2},$$

$$\chi=\frac{1}{2\eta_1}(\omega_2 z+x_0). \quad (36)$$

By solving Eqs. (34), we obtain

$$\bar{\theta}_s(x,z,\zeta)=\frac{\zeta-\bar{\zeta}_1}{\zeta-\zeta_1}I-\frac{2i\eta_1}{\zeta-\zeta_1}(I-F_1)(I-\bar{F}_1F_1)^{-1}, \quad (37)$$

when

$$\Omega(x,z)=4\eta_1(I-F_1)(I-\bar{F}_1F_1)^{-1}. \quad (38)$$

From relation (38), with help from relation (24), we obtain the solitonlike solution:

$$e(x,z)=\frac{2\eta_1(z)}{\beta}\exp[-i\vartheta(x,z)]\operatorname{sech} y(x,z). \quad (39)$$

The explicit expressions for the soliton matrices $\theta_s(x,\zeta)$ and $\bar{\theta}_s(x,\zeta)$ are (a) for the continuous spectrum:

$$\theta_s(x,\xi)=(\xi-\bar{\zeta}_1)^{-1}H[(y(x,z),\xi)],\quad \bar{\theta}_s(x,\xi)=(\xi-\zeta_1)^{-1}H[(y(x,z),\xi)],$$

$$H[(y(x,z),\xi)]=\begin{bmatrix}\xi-\xi_1-i\eta_1(z)\tanh y(x,z) & -i\eta_1(z)\exp[-i\vartheta(x,z)]\operatorname{sech} y(x,z)\\ -i\eta_1(z)\exp[i\vartheta(x,z)]\operatorname{sech} y(x,z) & \xi-\xi_1+i\eta_1(z)\tanh y(x,z)\end{bmatrix}. \quad (40)$$

and (b) for the discrete spectrum

$$\theta_s(x,\zeta_1)=\frac{1}{2}\begin{bmatrix}\exp[-y(x,z)] & -\exp[-i\vartheta(x,z)]\\ -\exp[i\vartheta(x,z)] & \exp[y(x,z)]\end{bmatrix}\operatorname{sech} y(x,z)$$

$$\bar{\theta}_s(x,\bar{\zeta}_1)=\frac{1}{2}\begin{bmatrix}\exp[y(x,z)] & \exp[-i\vartheta(x,z)]\\ \exp[i\vartheta(x,z)] & \exp[-y(x,z)]\end{bmatrix}\operatorname{sech} y(x,z). \quad (41)$$

III. SCATTERING DATA IN THE PRESENCE OF PERTURBATIONS

In practice, condition (4) might be only approximately accomplished, namely,

$$2\beta\nu=1+\varepsilon, \quad (42)$$

where the small parameter ε is due to the deviations of the light beam and medium features from those required in the soliton regime case. Let us consider the case when ε occurs as a consequence of the difference between the experimental value ν and ν_0 featuring the purely soliton regime, namely $2\beta\nu_0=1$ [e.g., $\varepsilon=2\beta(\nu-\nu_0)$].

In this case, the perturbation term appears in the right side of Eqs. (1) as

$$\frac{de}{dz}+i\frac{d^2e}{dx^2}-\nu_0\frac{de}{dx}+2i\beta^2|e|^2e+\langle\rho\rangle=(\varepsilon/2\beta)\frac{de}{dx},$$

$$\frac{d\rho}{dx}-2i\beta\delta\rho+2\beta en=-\varepsilon\frac{d\rho}{dx},$$

$$\frac{dn}{dx}-\beta(e\bar{\rho}+\bar{e}\rho)=-\varepsilon\frac{dn}{dx}. \quad (43)$$

For the perturbed system (43) we have the following ε curvature representation:

$$\frac{dU}{dz}-\frac{dV}{dx}+[U,V]=-i\left(\varepsilon\hat{R}+\frac{d\zeta}{dz}\sigma_3\right), \quad (44)$$

where the perturbation matrix $\hat{R}$ is of the form:

$$\hat{R}=\frac{1}{2}\begin{bmatrix}-\beta\left\langle\dfrac{\frac{dn}{dx}}{\zeta+\beta\delta}\right\rangle & i\dfrac{de}{dx}-\beta\left\langle\dfrac{\frac{d\rho}{dx}}{\zeta+\beta\delta}\right\rangle\\ -i\bar{e}-\beta\left\langle\dfrac{\frac{d\bar{\rho}}{dx}}{\zeta+\beta\delta}\right\rangle & \beta\left\langle\dfrac{\frac{dn}{dx}}{\zeta+\beta\delta}\right\rangle\end{bmatrix}. \quad (45)$$

The spectral equation (8) keeps its form, with the only specification that U_0 includes solutions of the perturbed equation (43), while the evolution equation (25) does not.

By using the soliton perturbation theory [10] formulated within the framework of the Riemann problem, we derive the evolution with z of the scattering matrix in the presence of the perturbations

$$\frac{dS}{dz}-[V_-,S]=-i\varepsilon\theta_+\left(\int_{-\infty}^{\infty}J^{-1}\theta^{-1}\hat{R}\theta J\,dx'\right)\theta_-^{-1}, \tag{46}$$

which gives

$$\frac{da}{dz}=-\varepsilon\langle\theta^{(1)}|\sigma_2\hat{R}|\theta^{(2)}\rangle_x,$$

$$\frac{db}{dz}+i(\omega_1+i\omega_2)b=\varepsilon\langle\theta^{(1)}|\sigma_2\hat{R}\exp(-2i\zeta x)|\overline{\theta}^{(1)}\rangle_x. \tag{47}$$

In this case $\theta^{(i)}$ are the columns of θ and the notation $\langle\theta^{(i)}|f(x)|\theta^{(j)}\rangle_x$ represents the integral

$$\int_{-\infty}^{\infty}\theta^{(i)}(x)f(x)\theta^{(j)}(x)dx. \tag{48}$$

The formulas for the discrete spectrum are obtained by making the substitution $\varepsilon\hat{R}\to\varepsilon\hat{R}+(d\zeta/dz)\sigma_3$ in relations (47), where the limit condition ζ approaches ζ_1 $(\zeta\to\zeta_1)$. Thus we get

$$\frac{d\grave{o}_1}{dz}=i\varepsilon\langle\theta_1^{(1)}|\sigma_2\hat{R}|\theta_1^{(2)}\rangle_x/\langle\theta_1^{(1)}|\sigma_1|\theta_1^{(2)}\rangle_x,$$

$$\frac{d\gamma_1}{dz}+i[\omega_1(\zeta_1)+i\omega_2(\zeta_1)]\gamma_1$$
$$=\varepsilon\frac{\gamma_1}{\dot{a}_1(\zeta_1)}\left[\left\langle\theta_1^{(2)}|\sigma_2\hat{R}|\frac{d}{d\zeta}[\theta^{(1)}(\zeta)-\gamma_1(x)\theta^{(2)}\right.\right.$$
$$\left.\left.\times(\zeta)]_{\zeta=\zeta_1}\right\rangle_x-2i\langle\theta_1^{(1)}|\sigma_2\hat{R}x|\theta_1^{(2)}\rangle_x\right], \tag{49}$$

where

$$\theta_1^{(i)}=\theta^{(i)}(x,\zeta_1). \tag{50}$$

IV. ADIABATIC APPROXIMATION

Let us consider the evolution of the soliton along the z axis in the adiabatic case. In this case we neglect the distortion of the soliton shape and the possible occurrence of a tail. Thus, the only consequence of the perturbation consists of a change of the z-axis evolution of the soliton parameters from that expressed by the relations (36).

Let us consider that at the boundary surface of the medium $(z=0)$, the electromagnetic field has the soliton form

$$e_s(x,z=0)=(2\eta_1/\beta)\exp\left[-2i\xi_1x-i\left(\varphi_0+\frac{\pi}{2}\right)\right]$$
$$\times\text{sech}(2\eta_1x-x_0). \tag{51}$$

We search the perturbed solution of $e(x,z)$ under the solitonlike form

$$e(x,z)=\frac{2\eta_1(z)}{\beta}\exp[-i\vartheta(x,z)]\text{sech}\,y(x,z), \tag{52}$$

where

$$y=2\eta_1(z)[x-\chi(z)],\quad \vartheta=[\xi_1(z)/\eta_1(z)]y+K(z). \tag{53}$$

The $\xi_1(z)$, $\eta_1(z)$, $\chi(z)$, and $K(z)$ are to be determined. The evolution of the soliton parameters is given by the relations (49).

In order to determine the perturbation matrix, we calculate the polarizability ρ and the population difference n using the solitontype solution (39) and the equations (1),

$$\rho=i\beta e\frac{\xi_1+\beta\delta+i\eta_1\tanh y}{(\xi_1+\beta\delta)^2+\eta_1^2},$$

$$n=-1+\frac{\beta^2}{2}\frac{|e|^2}{(\xi_1+\beta\delta)^2+\eta_1^2}. \tag{54}$$

When

$$R_{11}=4\beta\eta_1^3\left\langle\frac{1}{(\xi_1+\beta\delta)^2+\eta_1^2}\frac{1}{\zeta+\beta\delta}\right\rangle\text{sech}^2y\tanh y,$$

$$R_{12}=-ie\left\langle\frac{-K\,\text{sech}^2y+L\tanh y+M}{\zeta+\beta\delta}\right\rangle,$$

$$R_{21}=i\overline{e}\left\langle\frac{-K\,\text{sech}^2y+L\tanh y-M}{\zeta+\beta\delta}\right\rangle, \tag{55}$$

where

$$K=\frac{2i\beta^2\eta_1^2}{(\xi_1+\beta\delta)^2+\eta_1^2},$$

$$L=\eta_1(\xi_1+\beta\delta+i\eta_1)-\frac{\beta^3\delta\eta_1}{(\xi_1+\beta\delta)^2+\eta_1^2},$$

$$M=i\xi_1(\xi_1+\beta\delta+i\eta_1)-i\beta^2\frac{\xi_1^2+\eta_1^2+\beta\delta\xi_1}{(\xi_1+\beta\delta)^2+\eta_1^2}. \tag{56}$$

As referred to the z-axis evolution of the soliton parameters and making the direct calculation, we get $\langle\theta_1^{(1)}|\sigma_2\hat{R}|\theta_1^{(2)}\rangle_x=0$. It follows $\zeta_1=\xi_1+i\eta_1=\text{const}$.

Because $d\xi_1/dz=0$, we may consider $\xi_1=0$. This means a transform of the coordinates with respect to a referential frame tied up to the soliton wave. The parameters χ and K are given by the following equations:

$$\frac{d\chi}{dz}=\frac{\omega_2}{2\eta_1}+\frac{\varepsilon}{2\beta\eta_1}\text{Re}\left\langle(M-L)\frac{\beta\delta}{\beta^2\delta^2+\eta_1^2}\right\rangle$$
$$+\frac{\varepsilon}{2\beta}\text{Im}\left\langle\frac{2K-L+M}{\beta^2\delta^2+\eta_1^2}\right\rangle,$$

$$\frac{dK}{dz}=-\omega_1+\frac{\varepsilon}{\beta\eta_1}\text{Re}\left\langle(M-L)\frac{-\eta_1^2}{\beta^2\delta^2+\eta_1^2}\right\rangle$$
$$+\frac{\varepsilon}{\beta}\text{Im}\left\langle(2K-L+M)\frac{\beta\delta}{\beta^2\delta^2+\eta_1^2}\right\rangle. \tag{57}$$

For example, in the Lorentz line profile case

$$g(\delta)=(\Gamma/\pi)(\delta^2+\Gamma^2)^{-1}, \tag{58}$$

we get

$$\frac{dK}{dz}=-\omega_1, \tag{59}$$

$$\frac{d\chi}{dz}=\frac{1}{2\eta_1}\left\{\frac{\beta}{\Gamma\eta_1}\frac{1}{\Gamma+\eta_1/\beta}+\varepsilon\left[2\frac{2\Gamma+3\eta_1/\beta}{(\Gamma+\eta_1/\beta)^2}-\frac{\eta_1}{\beta}-\frac{1}{2}\Gamma\frac{\beta^2}{\eta_1^2}\right]\right\}\equiv\frac{1}{2\eta_1}(\omega_2+\varepsilon\Delta\omega_2). \tag{60}$$

Thus, for the Lorentz line profile and generally for any symmetrical function $g(\delta)$, the entire effect of the perturbation in the adiabatic case is present only in the expression of $\chi(z)$, while the amplitude of the soliton does not change,

$$e(x,z)=\frac{2\eta_1}{\beta}\exp\left(i\omega_1 z-i\varphi_0-i\frac{\pi}{2}\right)\times\operatorname{sech}[2\eta_1 x-(\omega_2+\varepsilon\Delta\omega_2)z-x_0] \tag{61}$$

V. CONCLUSIONS

In this paper we have studied the propagation of the light pulses in media with both resonant and nonresonant nonlinearities in order to establish the effect of such a symbiosis upon the evolution of optical pulses. We have shown that a slight deviation from the necessary condition for the solitons existence in such combined media leads to a modification of the soliton parameters. The established corrections were determined in an analytical way, within the framework of the perturbation theory, using a Riemann boundary problem in connection with the inverse scattering method. Thus, for the Lorentz profile line and generally for any symmetrical function $g(\delta)$ the entire effect of the perturbation in the adiabatic approximation appears only in the expression of the $\chi(z)$ parameter, while the amplitude of the soliton remains unchanged.

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